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Solving Generalized Fuzzy Transportation Problem Using New Strategies

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Abstract- Two new generalized fuzzy transportation models namely Generalized Fuzzy Cost Deviation Strategy and Next to Next Generalized Fuzzy Minimum Penalty Strategy are proposed in this paper for solving the generalized fuzzy transportation problems where the transportation cost, supply and demand are generalized trapezoidal fuzzy numbers. Both algorithms produce efficient initial solution in terms of minimum generalized fuzzy transportation cost and an optimal solution can be obtained by using Generalized fuzzy modified distribution method. Numerical examples are used to illustrate them and a comparison between the two proposed Strategies is made.

Keywords: Trapezoidal Fuzzy Numbers; Generalized Trapezoidal Fuzzy Numbers; Generalized Fuzzy Cost Deviation Strategy; Next to Next Generalized Fuzzy Minimum Penalty Strategy.

1. INTRODUCTION

The Transportation problem is the special type of linear programming problem where special mathematical structure of restrictions is used. Efficient algorithms have been developed for solving the transportation problem when the cost coefficients and the supply and demand quantities are known exactly. The occurrence of randomness and imprecision in the real world is inevitable owing to some unexpected situations. There are cases that the cost coefficients and the supply and demand quantities of a transportation problem may be uncertain due to some uncontrollable factors. To deal quantitatively with imprecise information in making decisions, Bellman and Zadeh (1970) and Zadeh(1965) introduced the notion of fuzziness.

Fuzzy transportation problem is a transportation problem whose decision parameters are fuzzy numbers. Lai and Hwang (1992) used tabular method for solving Fuzzy transportation problem by means of a crisp parametric programming problem. Stephen Dinagar (2009) investigated fuzzy transportation problem with the aid of trapezoidal fuzzy numbers and proposed fuzzy modified distribution method to find the optimal solution in terms of fuzzy numbers. Pandian and Natarajan (2010) proposed an algorithm namely fuzzy zero point method for finding an optimal solution to a fuzzy transportation problem.

Chen, (1985b) pointed out that in many cases it is not possible to restrict the membership function to the normal form and proposed the concept of generalized fuzzy numbers. There are several papers (Chen & Chen, (2003), Yong, et al., (2004), Chen& Chen, (2007), Chen & Chen, (2009), Chen & Wang, (2009)) in which generalized fuzzy numbers are used for solving real life problems.

In this paper, two new generalized fuzzy transportation methods namely Generalized Fuzzy Cost Deviation Strategy and Next to Next Generalized Fuzzy Minimum Penalty Strategy are proposed. Both methods are used for solving the generalized fuzzy transportation problems where the transportation cost, supply and demand are generalized trapezoidal fuzzy numbers. Both algorithms produce efficient initial solution in terms of minimum generalized fuzzy transportation cost. An optimal solution can be obtained by using Generalized fuzzy modified distribution method. Numerical examples are used to illustrate them and a comparison between the two proposed Strategies is made.

2. PRELIMINARIES

In this section basic definitions are reviewed according to Kaufmann and Gupta, (1988).

Definition 1:

The characteristic function of a crisp set A $\subseteq$ X assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}}$: X $\rightarrow$ [0,1]. The assigned value indicates the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set

$\tilde{A}$ = { $(x, \mu_{\tilde{A}}(x))$; $x \in$ X } defined by $\mu_{\tilde{A}}$ for each $x \in$X is called a fuzzy set.

Definition 2:

A fuzzy set $\tilde{A}$ defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\tilde{A}}$: R → [0,1] is continuous.

(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$

(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a, b]$and strictly decreasing on $[c, d]$.

(iv) $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b, c]$, where $a \leq b \leq c \leq d$.

Definition 3:

A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{(x-d)}{(c-d)}, & c \leq x \leq d \end{cases}$$

Definition 4:

A fuzzy set $\tilde{A}$ defined on the universal set of real numbers R, is said to be Generalized Fuzzy Number (GFN) if its membership function has the following characteristics

(i) $\mu_{\tilde{A}}$: R → [0,1] is continuous.

(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$

(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$.

(iv) $\mu_{\tilde{A}}(x) = w$, for all $x \in [b, c]$, where 0 ≤ w ≤ 1.

Definition 5:

A GFN $\tilde{A} = (a, b, c, d; w)$ is said to be a Generalized Trapezoidal Fuzzy Number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{w(x-a)}{(b-a)}, & a \leq x \leq b \\ w, & b \leq x \leq c \\ \frac{w(x-d)}{(c-d)}, & c \leq x \leq d \end{cases}$$

If in a Generalized Trapezoidal Fuzzy Number $\tilde{A}$, we let $b = c$ then we get a Generalized Triangular Fuzzy Number denoted by $\tilde{A} = (a, b, d; w)$.

Definition 6:

A GFN $\tilde{A} = (a, b, c, d; w)$ is said to be nonnegative Generalized Trapezoidal Fuzzy Number if and only if $a \geq 0$.

Definition 7:

Two GFNs $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ are said to be equal i.e., $\tilde{A} = \tilde{B}$ if only if $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$, $d_1 = d_2$, $w_1 = w_2$.

3. ARITHMETIC OPERATIONS OF GENERALIZED FUZZY NUMBERS

According to Chen& Chen, (2009), arithmetic operations between two generalized trapezoidal fuzzy numbers defined on universal set of real numbers R, are reviewed as follows

If $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ are two Generalized Trapezoidal Fuzzy numbers then

(i) Generalized Fuzzy numbers addition (⊕) :

$\tilde{A} \oplus \tilde{B} = \left(a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2;\ \text{minimum}(w_1, w_2)\right)$ where $a_1, b_1, c_1, d_1, a_2, b_2, c_2\ d_2$ are any real numbers.

(ii) Generalized Fuzzy numbers subtraction (⊖) :

$\tilde{A} \ominus \tilde{B} = \left(a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2;\ \text{minimum}(w_1, w_2)\right)$ where $a_1, b_1, c_1, d_1, a_2, b_2, c_2\ d_2$ are any real numbers.

(iii)Generalized Fuzzy numbers multiplication(⊗) :

$\tilde{A} \otimes \tilde{B} = \left(a', b', c', d';\ \text{minimum}(w_1, w_2)\right)$ where $a' = \text{minimum}(a_1a_2, a_1d_2, a_2d_1, d_1d_2)$ $b' = \text{minimum}(b_1b_2, b_1c_2, c_1b_2, c_1c_2)$ $c' = \text{maximum}\ (b_1b_2, b_1c_2, c_1b_2, c_1c_2)$ $d' = \text{maximum}\ (a_1a_2, a_1d_2, a_2d_1, d_1d_2)$ where $a_1, b_1, c_1, d_1, a_2, b_2, c_2\ d_2$ are any real numbers.

(iv)Generalized Fuzzy numbers scalar multiplication:

$$\lambda\tilde{A} = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1; w_1), \lambda > 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1; w_1), \lambda < 0 \end{cases}$$

4. RANKING OF GENERALIZED FUZZY NUMBER

Let Ã = (a , b , c ,d; w) be a generalized trapezoidal fuzzy number then

(i) Rank $R(\tilde{A}) = \frac{w(a+b+c+d)}{4}$,

(ii) mode $(\tilde{A}) = \frac{w(b+c)}{2}$,

(iii) divergence $(\tilde{A}) = w(d-a)$,

(iv) Left spread $(\tilde{A}) = w(b-a)$,

(v) Right spread $(\tilde{A}) = w(d-c)$

Two generalized trapezoidal fuzzy numbers $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ can be compared using the ranking functions given in Amit Kumar etal. , (2010).

5.GENERALIZEDFUZZY TRANSPORTATION PROBLEM (GFTP)

Consider a transportation problem with m generalized origins (rows) and n generalized fuzzy destinations (columns).

Let $\tilde{C}_{ij} = \left[c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}, w_{C_{ij}}\right]$ be the cost of transporting one unit of the product from i[th] generalized fuzzy origin to j[th] generalized fuzzy destination.

$\tilde{a}_i = \left[a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}\ w_{a_i}\right]$ be the quantity of commodity available at generalized fuzzy origin i. $\tilde{b}_j = \left[b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}\ w_{b_j}\right]$ be the quantity of commodity needed at generalized fuzzy destination j. $\tilde{X}_{ij} = \left[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}, w_{x_{ij}}\right]$ is the quantity transported from i[th] generalized fuzzy origin to j[th] generalized fuzzy destination.

The generalized fuzzy transportation problem can be stated in the tabular form as

	GFD$_1$	GFD$_2$		GFD$_n$	GFC
GFO$_1$	$\tilde{X}_{11}$ $\tilde{C}_{11}$	$\tilde{X}_{12}$ $\tilde{C}_{12}$	…	$\tilde{X}_{1n}$ $\tilde{C}_{1n}$	$\tilde{a}_1$
GFO$_2$	$\tilde{X}_{21}$ $\tilde{C}_{21}$	$\tilde{X}_{22}$ $\tilde{C}_{22}$	…	$\tilde{X}_{2n}$ $\tilde{C}_{2n}$	$\tilde{a}_2$
	⋮	⋮	⋮	⋮	
GFO$_m$	$\tilde{X}_{m1}$ $\tilde{C}_{m1}$	$\tilde{X}_{m2}$ $\tilde{C}_{m2}$	…	$\tilde{X}_{mn}$ $\tilde{C}_{mn}$	$\tilde{a}_m$
GFD	$\tilde{b}_1$	$\tilde{b}_2$		$\tilde{b}_n$	

where GFO$_i$ (i=1,2,…m) – Generalized Fuzzy Origin, GFD$_j$ (j=1,2,…n) – Generalized Fuzzy Destination, GFC – Generalized Fuzzy Capacity, GFD – Generalized Fuzzy Demand.

Definition 8 :

The given GFTP is said to be balanced if

$$\sum_{i=1}^{m}\left(a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}; w_{a_i}\right) = \sum_{j=1}^{n}\left(b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}; w_{b_j}\right)$$

(ie) if the total generalized fuzzy capacity is equal to the total generalized fuzzy demand.

Definition 9 :

Any set of generalized fuzzy allocations $\tilde{X}_{ij} \succ (-\lambda, -0.5\lambda, 0.5\lambda, \lambda; 0.5)$, $\lambda \in [0,1]$, which satisfies the row and column sum is a generalized fuzzy feasible solution.

6.GENERALIZED FUZZY COST DEVIATION STRATEGY

A Strategy, Generalized Fuzzy Cost Deviation Strategy is proposed to find the initial basic feasible solution of GFTP and the steps are summarized as follows:

Step 1: The generalized fuzzy cost deviation table is constructed for the given generalized fuzzy transportation problem as follows**.**

The row and column generalized fuzzy cost deviation of a cell for each row and each column is computed to obtain an ordered pair. This ordered pair is equal to the generalized fuzzy transportation cost of the cell minus the generalized fuzzy minimum of the corresponding row or column generalized fuzzy transportation cost. The ordered pair (θ,ϕ) is the generalized fuzzy cost deviation, if θ is the row generalized fuzzy cost deviation and ϕ is the column generalized fuzzy cost deviation of the cell.

Step 2: A row r, which contains the maximum generalized fuzzy row cost deviation is found.

Step 3: A column t which contains the maximum generalized fuzzy column cost deviation is found

Step 4: The minimum generalized fuzzy cost deviation in the rth row is found, say (a, b).

Step 5: The minimum generalized fuzzy cost deviation in the th column is found, say (c, d).

Step6: The corresponding cells for (a, b) and (c, d) are found say (x, y) and (α, β) respectively.

Step 7: (a) If $x \neq \alpha$ and $y \neq \beta$, both cells (x, y) and (α, β) are selected and the maximum possible is allocated to them.

(b) (i) If $x \neq \alpha$ and $y = \beta$ or $x = \alpha$ and $y \neq \beta$, if $(a, b) > (c, d)$, cell (x, y) is selected, the maximum possible is allocated to it. Then, cell (α, β) is selected and the maximum possible is allocated to it.

(ii) If $x \neq \alpha$ and $y = \beta$ or $x = \alpha$ and $y \neq \beta$, if $(a, b) < (c, d)$, cell (α, β) is selected, the maximum possible is allocated to it. Then, cell (x, y) is selected and the maximum possible is allocated to it.

(iii) If $x \neq \alpha$ and $y = \beta$ or $x = \alpha$ and $y \neq \beta$, if $(a, b) = (c, d)$, any one of the cell is selected and the maximum possible is allocated to it. Then, another cell is selected, the maximum possible is allocated to it.

(c) If $x = \alpha$ and $y = \beta$, cell $(a, b) = (c, d)$ is selected and the maximum possible is allocated to it.

Step 8: The generalized fuzzy transportation table is refined after deleting fully used generalized fuzzy supply points and fully received generalized fuzzy demand points. Incompletely used fully used generalized fuzzy supply points and incompletely received generalized fuzzy demand points are modified.

Step 9: The generalized fuzzy cost deviation table is constructed for the reduced generalized fuzzy transportation problem. Then, step 2 is executed.

Step 10 : The above process is repeated until all generalized fuzzy supply points are fully used or all fuzzy demand points are received.

Step 11 : A generalized fuzzy solution is resulted for the problem from the allotment.

7. NEXT TO NEXT GENERALIZED FUZZY MINIMUM PENALTY STRATEGY

A Strategy, Next to Next Generalized Fuzzy Minimum Penalty Strategy is proposed to find the initial basic feasible solution of GFTP and the steps are summarized as follows:

Step 1 : The smallest generalized fuzzy entry from the first row is chosen and it is subtracted from the third smallest generalized fuzzy entry. This value is written against the row on the right. This value is the generalized fuzzy penalty for the first row. Similarly, the generalized fuzzy penalty for each row is computed.

Step 2: Then the generalized fuzzy column penalties are calculated and they are written on the bottom of the generalized fuzzy cost matrix below their corresponding columns.

Step 3 : The highest generalized fuzzy penalty is selected and the row or column for which this corresponds is found. $Min\,(\tilde{a}_i, \tilde{b}_j)$ allocation is made to the cell having the lowest generalized fuzzy cost from the selected row or column.

Step 4 : The satisfied row or column is eliminated. Fresh generalized fuzzy penalties for the remaining generalized fuzzy sub matrix are calculated as in step 1 and step 2 and allocations are made as mentioned in step 3. This is continued until all the rim requirements are satisfied.

8. NUMERICAL EXAMPLE

In this section the methods discussed in sections 6 and 7 are illustrated by an example

Problem:

Consider a GFTP with rows representing three generalized fuzzy origins GFO_1 GFO_2 GFO_3 and columns representing four destinations GFD_1 , GFD_2, GFD_3, GFD_4.

	GFD_1	GFD_2	GFD_3	GFD_4	GFC
GFO_1	(2,3,4,5;0.2)	(2,4,5,7;0.5)	(1,2,3,4;0.1)	(2,4,6,8;0.6)	(1,4,6,10;0.2)
GFO_2	(2,3,6,13;0.3)	(1,2,3,6;0.1)	(2,3,6,14;0.1)	(2,6,11,15;0.3)	(2,4,6,8;0.2)
GFO_3	(2,6,10,17;0.2)	(2,6,8,12;0.2)	(1,2,4,6;0.1)	(1,3,7,10;0.1)	(5,10,14,20;0.2)
GFD	(2,6,9,14;0.2)	(3,6,8,10;0.2)	(2,3,4,7;0.2)	(1,3,5,7;0.2)	

Since $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ = (8,18,26,38;0.2), the problem is balanced GFTP and there exists a feasible solution to the GFTP.

Generalized Fuzzy Cost Deviation Strategy:

By step 1, the generalized fuzzy cost deviation table is constructed as

	GFD_1	GFD_2	GFD_3	GFD_4	GFC
GFO_1	[(-2,0,2,4;0.1), (-3,-1,1,3;0.2)]	[(-2,1,3,6;0.1), (-4,1,3,6;0.1)]	[(-3,-1,1,3;0.1), (-3,-1,1,3;0.1)]	[(-2,1,4,7;0.1), (-8,-3,3,7;0.1)]	(1,4,6,10;0.2)
GFO_2	[(-4,0,4,12;0.1), (-3,-1,3,11;0.2)]	[(-5,-1,1,5;0.1), (-5,-1,1,5;0.1)]	[(-4,0,4,13;0.1), (-2,0,4,13;0.1)]	[(-4,3,9,14;0.1), (-8,-1,8,14;0.1)]	(2,4,6,8;0.2)
GFO_3	[(-4,2,8,16;0.1), (-3,2,7,15;0.2)]	[(-4,2,6,11;0.1), (-4,3,6,11;0.1)]	[(-5,-2,2,5;0.1), (-3,-1,2,5;0.1)]	[(-5,-1,5,9;0.1), (-9,-4,4,9;0.1)]	(5,10,14,20;0.2)
GFD	(2,6,9,14;0.2)	(3,6,8,10;0.2)	(2,3,4,7;0.2)	(1,3,5,7;0.2)	

The maximum generalized fuzzy row cost deviation is identified, which is in cell (3,1) and the minimum generalized fuzzy row cost is selected from the third row , which is to be in cell (3,3). The maximum generalized fuzzy column cost deviation is identified, which is in cell (3,1) and the minimum generalized fuzzy column cost is selected from the third column , which is to be in cell (1,1). The selected cells are (3,3) , (1,1).

By step 7, maximum possible units are allocated to cells (3,3) and (1,1).

After the allocation is made on both the cells, it results in the generalized fuzzy transportation table shown below:

	GFD_1	GFD_2		GFD_4	GFC
GFO_2	(2,3,6,16;0.3)	(1,2,3,6,;0.1)		(2,6,11,15;0.3)	(2,4,6,8;0.2)
GFO_3	(2,6,10,17;0.2)	(2,6,8,12;0.2)		(1,3,7,10;0.1)	(-2,6,11,18;0.2)
GFD	(-8,0,5,13;0.2)	(3,6,8,10;0.2)		(1,3,5,7;0.2)	

Applying step 1 to the above table, the following generalized fuzzy cost deviation table is obtained.

	GFD_1	GFD_2		GFD_4	GFC
GFO_2	[(-4,0,4,12;0.1), (-15,-7,0,11;0.2)]	[(-5,-1,1,5;0.1), (-5,-1,1,5;0.1)]		[(-4,3,9,14;0.1), (-8,-1,8,14;0.1)]	(2,4,6,8;0.2)
GFO_3	[(-8,-1,7,16;0.1), (-15,-4,4,15;0.2)]	[(-8,-1,5,11;0.1), (-4,3,6,11;0.1)]		[(-9,-4,4,9;0.1), (-9,-4,4,9;0.1)]	(-2,6,11,18;0.2)
GFD	(-8,0,5,13;0.2)	(3,6,8,10;0.2)		(1,3,5,7;0.2)	

The maximum generalized fuzzy row cost deviation is identified, which is in cell (2,4) and the minimum generalized fuzzy row cost is selected from the second row , which is to be in cell (2,2). The maximum generalized fuzzy column cost deviation is identified, which is in cell (3,2) and the minimum generalized fuzzy column cost is selected from the second column , which is to be in cell (2,2). The selected cell is (2,2).
By step 7, maximum possible units are allocated to cells (2,2).
After the allocation is made on the cell, it results in the generalized fuzzy transportation table shown below:

	GFD_1	GFD_2		GFD_4	GFC
GFO_3	(2,6,10,17;0.2)	(2,6,8,12;0.2)		(1,3,7,10;0.1)	(-2,6,11,18;0.2)
GFD	(-8,0,5,13;0.2)	(-5,0,4,8;0.2)		(1,3,5,7;0.2)	

Applying step 1to the above table, the following generalized fuzzy cost deviation table is obtained.

	GFD_1	GFD_2		GFD_4	GFC
GFO_3	[(-8,-1,7,16;0.1), (2,6,10,17;0.2)]	[(-8,-1,5,11;0.1), (2,6,8,12;0.2)]		[(-9,-4,4,9;0.1) (1,3,7,10;0.1)]	(-2,6,11,18;0.2)
GFD	(-8,0,5,13;0.2)	(-5,0,4,8;0.2)		(1,3,5,7;0.2)	

The maximum generalized fuzzy row cost deviation is identified, which is in cell (3,1) and the minimum generalized fuzzy row cost is selected from the second row , which is to be in cell (3,4). The maximum generalized fuzzy column cost deviation is identified, which is in cell (3,1) and the minimum generalized fuzzy column cost is selected from the second column , which is to be in cell (3,1). The selected cells are (3,4) and (3,1).
By step 7, maximum possible units are allocated to cells (3,4) and (3,1).
After the allocation is made on the cells, the allocation is completed as follows

	GFD_1	GFD_2	GFD_3	GFD_4	GFC
GFO_1	**(1,4,6,10;0.2)** (2,3,4,5;0.2)	(2,4,5,7;0.5)	(1,2,3,4;0.1)	(2,4,6,8;0.6)	(1,4,6,10;0.2)
GFO_2	(2,3,6,13;0.3)	**(2,4,6,8;0.2)** (1,2,3,6,;0.1)	(2,3,6,14;0.1)	(2,6,11,15;0.3)	(2,4,6,8;0.2)
GFO_3	**(-8,0,5,13;0.2)** (2,6,10,17;0.2)	**(-5,0,4,8;0.2)** (2,6,8,12;0.2)	**(2,3,4,7;0.2)** (1,2,4,6;0.1)	**(1,3,5,7;0.2)** (1,3,7,10;0.1)	(5,10,14,20;0.2)
GFD	(2,6,9,14;0.2)	(3,6,8,10;0.2)	(2,3,4,7;0.2)	(1,3,5,7;0.2)	

Initial basic feasible solution to the given GFTP is obtained and the generalized fuzzy transportation cost according to the above route is
$(1,4,6,10;0.2)\otimes(2,3,4,5;0.2)\oplus$
$(2,4,6,8;0.2)\otimes(1,2,3,6;0.1)\oplus$
$(-8,0,5,13;0.2)\otimes(2,6,10,17;0.2)\oplus$
$(-5,0,4,8;0.2)\otimes(2,6,8,12;0.2)\oplus$
$(2,3,4,7;0.2)\otimes(1,2,4,6;0.1)\oplus$
$(1,3,5,7;0.2)\otimes(1,3,7,10;0.1)$
$= (-189, 35,175,527;0.1)$
The optimal generalized fuzzy basic feasible solution by GFMDM (Sagaya and Henry,(2011)) is obtained as follows
By step1, assign $\tilde{u}_3 = (-1,-0.5,0.5,1;0.5)$ to the third row (the row with maximum number of allocation). Thus,
$\tilde{v}_1 =(2,6,10,17;0.2)\ominus(-1,-0.5,0.5,1;0.5) = (1,5.5,10.5,18;0.2)$
$\tilde{v}_2 =(2,6,8,12;0.2)\ominus(-1,-0.5,0.5,1;0.5)= (1,5.5,8.5,13;0.2)$
$\tilde{v}_3 =(1,2,4,6;0.1)\ominus(-1,-0.5,0.5,1;0.5)= (0,1.5,4.5,7;0.1)$
$\tilde{v}_4 =(1,3,7,10;0.1)\ominus(-1,-0.5,0.5,1;0.5)= (0,2.5,7.5,11;0.1)$
$\tilde{u}_2 = (1,2,3,6;0.1) \ominus(1,5.5,8.5,13;0.2)= (-12,-6.5,-2.5,5;0.1)$
$\tilde{u}_1 =(2,3,4,5;0.2)\ominus(1,5.5,10.5,18;0.2)= (-16,-7.5,-1.5,4;0.2)$
The net evaluations for each of the unoccupied cells are determined as follows :
$\tilde{z}_{12} \ominus \tilde{c}_{12} = (\tilde{u}_1 \oplus \tilde{v}_2) \ominus \tilde{c}_{12} = (-22,-7,3,15;0.2)$
$\tilde{z}_{13} \ominus \tilde{c}_{13} = (\tilde{u}_1 \oplus \tilde{v}_3) \ominus \tilde{c}_{13} = (-20,-9,1,10;0.1)$
$\tilde{z}_{14} \ominus \tilde{c}_{14} = (\tilde{u}_1 \oplus \tilde{v}_4) \ominus \tilde{c}_{14} = (-24,-11,2,13;0.1)$
$\tilde{z}_{21} \ominus \tilde{c}_{21} = (\tilde{u}_2 \oplus \tilde{v}_1) \ominus \tilde{c}_{21} = (-24,-7,5,21;0.1)$

$\tilde{z}_{23} \ominus \tilde{c}_{23} = (\tilde{u}_2 \oplus \tilde{v}_3) \ominus \tilde{c}_{23} =$ (-26,-11,-1,10;0.1)

$\tilde{z}_{24} \ominus \tilde{c}_{24} = (\tilde{u}_2 \oplus \tilde{v}_4) \ominus \tilde{c}_{24} =$ (-27,-15,-1,14;0.1)

Since all $\left(z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}, z_{ij}^{(4)}; w_{z_{ij}}\right) \ominus \left(c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}; w_{c_{ij}}\right) \prec (-1, -0.5, 0.5, 1; 0.5)$ the current basic feasible solution is an optimum one and the optimal generalized fuzzy transportation cost associated with the optimum schedule is $\left(Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}; w_Z\right)$ =(-189,35,175,527;0.1)

Next to Next Generalized Fuzzy Minimum Penalty Strategy :

By step 1and step 2, the generalized fuzzy row penalty for each row and the generalized fuzzy column penalty for each column are computed and written against the row on the right
and on the bottom of the generalized fuzzy cost matrix below their corresponding columns.
And the resulting table is shown below

	GFD$_1$	GFD$_2$	GFD$_3$	GFD$_4$	GFC	Penalties
GFO$_1$	(2,3,4,5;0.2)	(2,4,5,7;0.5)	(1,2,3,4;0.1)	(2,4,6,8;0.6)	(1,4,6,10;0.2)	(-2,1,3,6;0.1)
GFO$_2$	(2,3,6,13;0.3)	(1,2,3,6,;0.1)	(2,3,6,14;0.1)	(2,6,11,15;0.3)	(2,4,6,8;0.2)	(-4,0,4,12;0.1)
GFO$_3$	(2,6,10,17;0.2)	(2,6,8,12;0.2)	(1,2,4,6;0.1)	(1,3,7,10;0.1)	(5,10,14,20;0.2)	(-4,2,6,11;0.1)
GFD	(2,6,9,14;0.2)	(3,6,8,10;0.2)	(2,3,4,7;0.2)	(1,3,5,7;0.2)		
Penalties	(-3,-1,3,11;0.2)	(-4,1,3,6;0.1)	(-2,0,4,13;0.1)	(-8,-3,3,7;0.1)		

In the table above, the maximum generalized fuzzy penalty is found in the first column. So, the maximum possible unit (1,4,6,10;0.2) is allocated to the minimum cost cell i.e.,(1,1). The remaining stock is written in the first column. The first row is removed and steps 1 and 2 are repeated. The resulting table is shown below :

	GFD$_1$	GFD$_2$	GFD$_3$	GFD$_4$	GFC	Penalties
GFO$_2$	(2,3,6,13;0.3)	(1,2,3,6,;0.1)	(2,3,6,14;0.1)	(2,6,11,15;0.3)	(2,4,6,8;0.2)	(-4,0,4,12;0.1)
GFO$_3$	(2,6,10,17;0.2)	(2,6,8,12;0.2)	(1,2,4,6;0.1)	(1,3,7,10;0.1)	(5,10,14,20;0.2)	(-4,2,6,11;0.1)
GFD	(-8,0,5,13;0.2)	(3,6,8,10;0.2)	(2,3,4,7;0.2)	(1,3,5,7;0.2)		
Penalties	(-15,-7,0,11;0.2)	(-4,3,6,11;0.1)	(-4,-1,4,13;0.1)	(-8,-1,8,14;0.1)		

In the table above, the maximum generalized fuzzy penalty is found in the second column. So, the maximum possible unit (2,4,6,8;0.2) is allocated to the minimum cost cell i.e.,(2,2). The remaining stock is written in the second column. The second row is removed and steps 1 and 2 are repeated. The resulting table is shown below :

	GFD$_1$	GFD$_2$	GFD$_3$	GFD$_4$	GFC	Penalties
GFO$_3$	(2,6,10,17;0.2)	(2,6,8,12;0.2)	(1,2,4,6;0.1)	(1,3,7,10;0.1)	(5,10,14,20;0.2)	(-4,2,6,11;0.1)
GFD	(-8,0,5,13;0.2)	(-5,0,4,8;0.2)	(2,3,4,7;0.2)	(1,3,5,7;0.2)		
Penalties	(2,6,10,17;0.2)	(2,6,8,12;0.2)	(1,2,4,6;0.1)	(1,3,7,10;0.1)		

Repeating the steps 3 and 4, the resulting allocation table is as follows

	GFD$_1$	GFD$_2$	GFD$_3$	GFD$_4$	GFC
GFO$_1$	**(1,4,6,10;0.2)** (2,3,4,5;0.2)	(2,4,5,7;0.5)	(1,2,3,4;0.1)	(2,4,6,8;0.6)	(1,4,6,10;0.2)
GFO$_2$	(2,3,6,13;0.3)	**(2,4,6,8;0.2)** (1,2,3,6,;0.1)	(2,3,6,14;0.1)	(2,6,11,15;0.3)	(2,4,6,8;0.2)
GFO$_3$	**(-8,0,5,13;0.2)** (2,6,10,17;0.2)	**(-5,0,4,8;0.2)** (2,6,8,12;0.2)	**(2,3,4,7;0.2)** (1,2,4,6;0.1)	**(1,3,5,7;0.2)** (1,3,7,10;0.1)	(5,10,14,20;0.2)
GFD	(2,6,9,14;0.2)	(3,6,8,10;0.2)	(2,3,4,7;0.2)	(1,3,5,7;0.2)	

Initial basic feasible solution to the given GFTP is obtained and the generalized fuzzy transportation cost according to the above route is = (-189, 35,175,527;0.1)

By applying GFMDM (Sagaya and Henry,(2011)), it is found that the current basic feasible solution is an optimum one and the optimal generalized fuzzy transportation cost associated with the optimum schedule is $\left(Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}; w_Z\right)$ =(-189,35,175,527;0.1)

The Initial basic feasible solution to the given GFTP obtained by both the stragegies Generalized Fuzzy Cost Deviation Strategy and Next to Next Generalized Fuzzy Minimum Penalty Strategy is the same and the optimal generalized fuzzy transportation cost associated with the optimum schedule is $\left(Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}; w_Z\right)$ =(-189,35,175,527;0.1)

9. CONCLUSION

Generalized Fuzzy Cost Deviation Strategy and Next to Next Generalized Fuzzy Minimum Penalty Strategy are proposed to find the initial basic feasible solution of the generalized fuzzy transportation problem where transported costs, the supply quantities and the demand quantities are generalized trapezoidal fuzzy numbers. These Strategies are systematic procedures, both easy to understand and to apply and their solution is close to the optimum solution.

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